Relativistic effects in radiative transitions of charmonia

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In the framework of a relativistic quark model constructed in an infinite momentum frame, corrections of the order of v^2/c^2 are obtained in the formulas which express amplitudes of the transitions $\psi(\psi') \to \eta_c(\eta_c')\gamma$, $\chi_J \to \psi\gamma$, $\psi' \to \chi_J\gamma$, $h_c \to \eta_c\gamma$, $\eta_c' \to h_c\gamma$ through overlap integrals $\int \phi_f^*(q)\phi_i(q)d\mathbf{q}$, $\int \phi_f^*(r)r\phi_i(r)d\mathbf{r}$. These corrections lead to a considerable suppression of $\Gamma(\psi \to \eta_c\gamma)$, however, they are insufficient to remove the existing disagreement of quark model predictions with experiment. Taking into account the relativistic corrections, the ratios of overlap integrals for $\chi_0 \to \psi\gamma$, $\chi_2 \to \psi\gamma$ and $\psi' \to \chi_0\gamma$, $\psi' \to \chi_2\gamma$ transitions are extracted from experiment; the obtained results are compared with quark model predictions.

1 Introduction

In addition to the mass spectrum of $q\bar{q}$ - states, which is a good laboratory for understanding of interquark forces, the quarkonium single-photon transitions also provide sensitive tests for potential models. Recently a considerable progress has been achieved in the experimental investigation of the widths of charmonium radiative transitions due to the more precise measurement of the ψ width [1] and due to the first measurements of the χ_{c1} and χ_{c2} widths [2] as well. The further progress in the investigation of the charmonium radiative transitions will be available at tau-charm factories where along with more precise measurements of the $\chi_J \to \psi\gamma$, $\psi' \to \chi_J\gamma$, $\psi \to \eta_c\gamma$ decay widths, the measurements of the radiative transitions of the missing η_c' and recently discovered ${}^1P_1(h_c)$ [3] states ($\psi' \to \eta_c'\gamma$, $h_c \to \eta_c\gamma$, $\eta_c' \to h_c\gamma$) can be accessed.

The present work is devoted to studying the relativistic effects in the above-listed charmonium radiative transitions, namely, the corrections of the order of v^2/c^2 will be found in the formulas which express amplitudes of these transitions through the wave-function overlap integrals:

$$I_{fi} = \int \phi_f^*(q)\phi_i(q)d\mathbf{q}, \quad r_{fi} = \int \phi_f^*(r)r\phi_i(r)d\mathbf{r}. \tag{1}$$

The reasons, why such consideration is important, are following. (a) The relativistic effects in wave functions of charmonia are investigated in many papers (see, e.g., refs.[4-6]) within a two-component model for interquark interactions: short distance one-gluon exchange and long-range confinement; however, to calculate the amplitudes of radiative transitions in these papers formulas of nonrelativistic quantum mechanics are used. (b) In the quark model predictions for the overlap integrals (1) there are results, which depend weakly on the details in choosing of the potential and on the method of taking into account the relativistic corrections in the wave functions. These are ratios of the overlap integrals r_{fi} for different states χ_J, which are close to the unity due to the small spin-orbit splitting of χ_0, χ_1, χ_2. Moreover, the analysis of the results of ref.[4,6] shows that the deviations of these ratios from the unity are hopefully predicted in the quark model. These are the overlap integrals I_{fi} for the transitions $\psi \to \eta_c\gamma$, $\psi' \to \eta_c'\gamma$, which are equal to the unity in nonrelativistic approximation; the relativistic corrections do not considerably affect these integrals due to the small spin-spin splitting generated by the one-gluon exchange in the $L = 0$ charmonium systems. The ratios of the overlap integrals r_{fi} for the transitions $h_c \to \eta_c\gamma$ and $\chi_J \to \psi\gamma$, $\eta_c' \to h_c\gamma$ and $\psi' \to \chi_J\gamma$ are close to the unity too, since the absence of significant long-range spin-spin forces, as is seen in the near degeneracy of h_c and the χ_J multiplet c.o.g., leads to near equality of the wave functions for h_c and χ_J. Therefore, for a comparison of these quantities with experiment it is especially important to take into account correctly relativistic effects in the amplitudes.

The relativistic corrections in the amplitudes of charmonium radiative transitions, we will investigate within a relativistic quark model constructed in the light-front

dynamics in refs.[7,8] and formulated later in an infinite-momentum frame (IMF) in refs.[9,10]. It turned out that in the considered approximation (when only corrections of the order of v^2/c^2 have been taken into account) the amplitudes of the transitions under interest are determined by the form factors which can be found using only linear in photon transverse momentum terms of the longitudinal components of electromagnetic transition current. In ref.[8] it was shown, that the results obtained using these terms are consistent with relativistic invariance.

2 Basic formulas and results

The relativistic quark model [9,10] is constructed for the hadron radiative transitions $A(P) \to B(P') + \gamma^*(K)$ (in parentheses the momenta of particles are given) in the IMF, where the initial hadron moves along the z axis with momentum $P \to \infty$, and the photon momentum components are $\mathbf{K}_\perp$, $K_z = -(m_A^2 - m_B^2 - \mathbf{K}_\perp^2)/4P$, $K_0 = -K_z$, $K^2 = -\mathbf{K}_\perp^2$. In this frame (see also ref.[11]), the space-time picture of the process for the longitudinal components of the electromagnetic current taken between hadron states with the helicities: $\lambda = \pm S_A$, $\lambda' = \pm S_B$, is the same as in the nonrelativistic quantum mechanics. The corresponding matrix elements have the following form [9,10]:

$$\frac{1}{2P}\langle P', \lambda' = \pm S_B \mid J_{0,z} \mid P, \lambda = \pm S_A\rangle \mid_{P\to\infty} = \frac{4}{3} e \int d\Gamma \Phi_A(M_0^2)\Phi_B(M_0'^2)T, \quad (2)$$

$$T = Sp\{U^+(x_{\bar c}, \mathbf{q}'_{\bar c_\perp})(\Gamma_B^{\lambda'})^+ U(x_c, \mathbf{q}'_{c_\perp}) U^+(x_c, \mathbf{q}_{c_\perp})\Gamma_A^\lambda U(x_{\bar c}, \mathbf{q}_{\bar c_\perp})\}. \quad (3)$$

Here we have assumed, that the hadrons A,B are bound states $c\bar c$, the quark momenta in the initial and final hadrons are parametrized in the IMF by:

$$\begin{aligned} &\mathbf{q}_i = x_i \mathbf{P} + \mathbf{q}_{i_\perp}, \quad \mathbf{q}'_i = x_i \mathbf{P}' + \mathbf{q}'_{i_\perp}, \ i = c, \bar c, \\ &x_c = 1 - x, \quad x_{\bar c} = x, \quad \mathbf{q}_{c\perp} = -\mathbf{q}_{\bar c\perp} = -\mathbf{q}_\perp, \\ &\mathbf{q}'_{c\perp} = -\mathbf{q}'_{\bar c\perp} = -\mathbf{q}'_\perp, \quad \mathbf{q}'_\perp = \mathbf{q}_\perp + x\mathbf{K}_\perp, \end{aligned} \quad (4)$$

M_0 and M_0' are invariant masses of the systems of initial and final quarks:

$$M_0^2 = \frac{m^2 + \mathbf{q}_\perp^2}{x(1-x)}, \quad M_0'^2 = \frac{m^2 + \mathbf{q}'^2_\perp}{x(1-x)}, \quad (5)$$

m is the mass of the c quark. The introduced variables are related to the 4-momentum of the initial quarks in their CMS $q(\mathbf{q}_\perp, q_z, \varepsilon)$ by:

$$q_z + \varepsilon = M_0 x, \ \varepsilon^2 = \mathbf{q}^2 + m^2, \ M_0 = 2\varepsilon. \quad (6)$$

In terms of variables x, $\mathbf{q}_\perp$ and $\mathbf{q}$ the phase space volume $d\Gamma$ has the form:

$$(2\pi)^3 d\Gamma = dx \ d\mathbf{q}_\perp / 2x(1-x) = 2d\mathbf{q}/M_0. \quad (7)$$

In eqs. (2,3) we have taken into account, in accordance with the results of refs. [9,10], that the vertex functions of the hadron transitions to quarks in the IMF are related to the wave functions of quarks in their CMS by spin rotation given by the Melosh matrix:

$$U(x_i, \mathbf{q}_{i_\perp}) = \frac{m + M_0 x_i + i\varepsilon_{lm}\sigma_l q_{im}}{[(m+M_0x_i)^2 + \mathbf{q}_\perp{}^2]^{1/2}}; \tag{8}$$

Γ are the spin-orbit parts of these wave functions in the CMS of quarks:

$$\Gamma_{\eta_c} = \frac{i}{\sqrt{2}}, \quad \Gamma^\lambda_\psi = \sigma \mathbf{e}_\psi^{(\lambda)}/\sqrt{2}, \qquad \Gamma_{h_c} = \sqrt{\frac{3}{2}}\mathbf{n}\mathbf{e}_{h_c}^{(\lambda)}, \tag{9}$$

$$\Gamma_{\chi_0} = \sigma\mathbf{n}/\sqrt{2}, \quad \Gamma^\lambda_{\chi_1} = \frac{\sqrt{3}}{2}[\sigma\mathbf{n}]\mathbf{e}^{(\lambda)}_{\chi_1}, \quad \Gamma^\lambda_{\chi_2} = \sqrt{\frac{3}{2}}(e^{(\lambda)}_{\chi_2})_{lm} n_l \sigma_m, \tag{10}$$

($\mathbf{n} = \mathbf{q}/q$). $\Phi(M_0^2)$ are radial parts of wave functions which in the present work we take in the form corresponding to the oscillator potential:

$$\begin{aligned}
&\Phi_{\psi(\eta_c)}(M_0^2)/\sqrt{M_0} \sim exp(-M_0^2/4\alpha^2) \sim exp(-q^2/\alpha^2),\\
&\Phi_{\chi(h_c)}(M_0^2)/\sqrt{M_0} \sim q\ exp(-M_0^2/4\alpha^2),\\
&\Phi_{\psi'(\eta_c')}(M_0^2)/\sqrt{M_0} \sim (q^2 - \frac{3}{4}\alpha^2)exp(-M_0^2/4\alpha^2).
\end{aligned} \tag{11}$$

This permits us to make calculations analytically and obtain the contribution of relativistic corrections into transition amplitudes in an explicit form.

The widths of the considered decays are related to the amplitudes corresponding to the M1 and E1 transitions via:

$$\Gamma(\psi(\psi') \to \eta_c(\eta_c')\gamma) = \alpha\ \frac{16}{27}\ \frac{\omega^3}{m^2}|M1|^2, \tag{12}$$

$$\Gamma(\chi_J \to \psi\gamma) = \alpha\ \frac{16}{81}\ \omega^3|E1|^2, \tag{13}$$

$$\Gamma(\psi' \to \chi_J\gamma) = \alpha\ \frac{16}{81}\ \frac{2J+1}{3}\ \omega^3|E1|^2, \tag{14}$$

$$\Gamma(h_c \to \eta_c\gamma) = \alpha\ \frac{16}{81}\ \omega^3|E1|^2, \tag{15}$$

$$\Gamma(\eta_c' \to h_c\gamma) = \alpha\ \frac{16}{27}\ \omega^3|E1|^2. \tag{16}$$

In the nonrelativistic approximation these amplitudes are equal directly to the wave-function overlap integrals: $M1(nonrel) = I_{fi}$, $E1(nonrel) = r_{fi}$. Let us note, that the functions $\phi(r)$ and $\phi(q) \sim \Phi(M_0^2)/\sqrt{M_0}$ are radial parts of wave functions in the momentum and coordinate representations, which are normalized by:

$$\int |\phi(r)|^2 d\mathbf{r} = \int |\phi(q)|^2 d\mathbf{q} = \int |\Phi(M_0^2)|^2 d\Gamma = 1. \tag{17}$$

In order to take into account the relativistic effects, one should find relativistic corrections in formulas expressing the amplitudes M1 and E1 through the overlap integrals I_{fi} and r_{fi}. In addition, the contributions corresponding to M2 transition in the $\chi_1 \to \psi\gamma$, $\psi' \to \chi_1\gamma$ decays and to M2 and E3 transitions in the $\chi_2 \to \psi\gamma$, $\psi' \to \chi_2\gamma$ decays, should be taken into account. These contributions are suppressed as compared to the contribution of E1 as ω^2/m^2, where $\omega = (m_i^2 - m_f^2)/2m_i$. For the oscillator potential, this is the value of the order of $(\alpha/m)^4$. Therefore, in the considered approximation (when we take into account only corrections of the order of $v^2/c^2 \sim \alpha^2/m^2$) these contributions should be neglected. In this approximation amplitudes M1 and E1 are equal to the form factors corresponding to the following relativistic - covariant expressions for the matrix elements of the electromagnetic current:

$$\psi \to \eta_c\gamma: \qquad \langle P'|J_\mu|P,\ \lambda\rangle = \ e\ \tfrac{4}{3}M1\ \varepsilon_{\mu\nu\sigma\rho}K^\nu P^\sigma\ e^\rho_{(\lambda)}, \tag{18}$$

$$h_c \to \eta_c\gamma: \qquad \langle P'|J_\mu|P,\ \lambda\rangle = \ e\ \tfrac{4}{3\sqrt{3}}E1\ [e^{(\lambda)}_\mu(PK)\ -\ P_\mu(Ke^{(\lambda)})], \tag{19}$$

$$\chi_0 \to \psi\gamma: \qquad \langle P',\ \lambda'|J_\mu|P\rangle = \ e\ \tfrac{4}{9}E1\ [e^{(\lambda')}_\mu(PK)\ -\ P_\mu(Ke^{(\lambda')})], \tag{20}$$

$$\chi_1 \to \psi\gamma: \qquad \langle P',\ \lambda'|J_\mu|P,\ \lambda\rangle = \ e\ \sqrt{\tfrac{32}{27}}\ \tfrac{m_\psi m_\chi}{m_\psi + m_\chi}E1\ \varepsilon_{\mu\nu\sigma\rho}K^\nu e^\sigma_{(\lambda')}\ e^\rho_{(\lambda)}, \tag{21}$$

$$\chi_2 \to \psi\gamma: \quad \langle P',\ \lambda'|J_\mu|P,\ \lambda\rangle = \ e\ \tfrac{4}{3\sqrt{3}}E1\ [\ e^{(\lambda)}_{\mu\nu}(KP) - P_\mu e^{(\lambda)}_{\nu\alpha}K^\alpha]\ e^\nu_{(\lambda')}. \tag{22}$$

It can be easily seen that all these form factors, except that for the $\chi_1 \to \psi\gamma$ transition, can be found using the relations (2,3) from the following matrix elements:

$$\frac{1}{2P}\langle\lambda_{\eta_c} = 0|J_0|\lambda_\psi = 1\rangle = ie\ \frac{\sqrt{2}}{3}(M1)K_x, \tag{23}$$

$$\frac{1}{2P}\langle\lambda_{\eta_c} = 0|J_0|\lambda_{h_c} = 1\rangle = e\ \sqrt{\frac{2}{3}}\ \frac{E1}{3}K_x, \tag{24}$$

$$\frac{1}{2P}\langle\lambda_\psi = 1|J_0|\lambda_{\chi_0} = 0\rangle = e\ \frac{\sqrt{2}}{9}(E1)K_x, \tag{25}$$

$$\frac{1}{2P}\langle\lambda_\psi = 1|J_0|\lambda_{\chi_2} = 2\rangle = e\ \sqrt{\frac{2}{3}}\ \frac{E1}{3}K_x. \tag{26}$$

The amplitude E1 for the $\chi_1 \to \psi\gamma$ transition can not be found from relations (2,3) in the leading over P order; therefore, in the considered approach we can not obtain reliable results for this transition. From the relations (23-26) it is seen, that all $M1$ and $E1$ amplitudes for remaining transitions are determined by the linear over K_x terms of the matrix elements (2). To find these terms, one should keep in $\Phi_B(M_0'^{\,2})$ only the terms of the order of $(K_x)^0$ and K_x:

$$\Phi_B(M_0'^{\,2}) = \Phi_B(M_0^2)\left[1 - \frac{q_x K_x}{(1 - q_z/m)\alpha^2} + \frac{q_x K_x}{4m^2}\right], \tag{27}$$

and write the quantity T in the form:

$$T = Sp\{I_{\bar{c}} \left(\Gamma^{\lambda'}_{B}\right)^{+} I_c \Gamma^{\lambda}_{A}\},$$

where

$$I_c \equiv U(x_c, \mathbf{q}'_{c\perp}) U^{+}(x_c, \mathbf{q}_{\mathbf{c}\perp}) = 1 + i\frac{K_x}{4m}\left[\sigma_2\left(1 + \frac{q_z}{2m} - \frac{2q^2}{3m^2}\right) + \sigma_3 \frac{q_y}{2m}\right], \tag{28}$$

$$I_{\bar{c}} \equiv U(x_{\bar{c}}, \mathbf{q}_{\bar{c}\perp}) U^{+}(x_{\bar{c}}, \mathbf{q}'_{\bar{\mathbf{c}}\perp}) = 1 + i\frac{K_x}{4m}\left[\sigma_2\left(1 + \frac{3q_z}{2m} - \frac{q^2}{3m^2}\right) + \sigma_3 \frac{q_y}{2m}\right]. \tag{29}$$

Further, using the formulas (2,3,23-29), it is easy to obtain:

$$M1(\psi \to \eta_c\gamma) = I_{\eta_c\psi}\left(1 - \frac{3\alpha^2}{8m^2}\right), \tag{30}$$

$$M1(\psi' \to \eta'_c\gamma) = I_{\eta'_c\psi'}\left(1 - \frac{33\alpha^2}{56m^2}\right), \tag{31}$$

$$E1(h_c \to \eta_c\gamma) = r_{\eta_c h_c}, \quad E1(\chi_0 \to \psi\gamma) = r_{\psi\chi_0}, \tag{32}$$

$$E1(\chi_2 \to \psi\gamma) = r_{\psi\chi_2}\left(1 + \frac{\alpha^2}{4m^2}\right), \tag{33}$$

$$E1(\eta'_c \to h_c\gamma) = r_{h_c\eta'_c}\left(1 - \frac{7}{12}\,\frac{\alpha^2}{m^2}\right), \tag{34}$$

$$E1(\psi' \to \chi_0\gamma) = r_{\chi_0\psi'}\left(1 - \frac{7}{12}\,\frac{\alpha^2}{m^2}\right), \tag{35}$$

$$E1(\psi' \to \chi_2\gamma) = r_{\chi_2\psi'}\left(1 - \frac{5}{6}\,\frac{\alpha^2}{m^2}\right). \tag{36}$$

3 Comparison with experiment and discussion

To demonstrate the role of the relativistic effects in the amplitudes of the $\chi_0 \to \psi\gamma$, $\chi_2 \to \psi\gamma$ and $\psi' \to \chi_0\gamma$, $\psi' \to \chi_2\gamma$ transitions we have presented in table 1 the ratios of the overlap integrals for these transitions extracted from experiment [12] using the nonrelativistic formulas and the formulas (32,33), (35,36). It is seen that the relativistic corrections change the ratios $r_{\psi\chi_0}/r_{\psi\chi_2}$ and $r_{\chi_0\psi'}/r_{\chi_2\psi'}$ in opposite directions. This seems to be right from the point of view of agreement between the quark model predictions [4,6] and experiment. However, the experimental errors are large and for strict conclusions new measurements are necessary.

The results for the $\psi \to \eta_c\gamma$ transition are presented in table 2. The typical for the potential models value: $m = 1.6$ GeV [4-6,13-15], is used for the c-quark mass. To demonstrate the dependence on this mass the results obtained with $m = 1.8$ GeV are also presented. For α^2/m^2 the value giving the correct mean magnitude of q^2/m^2

in ψ: $\alpha^2/m^2 = \frac{4}{3}\langle q^2/m^2\rangle_\psi \cong 0.3$ [13-15] is taken. It is seen that the relativistic corrections act in right direction leading to a suppression of $\Gamma(\psi \to \eta_c\gamma)$; however, they are insufficient to remove the existing disagreement of the nonrelativistic quark model prediction with experiment. The disagreement can be removed, if we take larger c-quark masses, $m > 1.8\ GeV$, which can be accommodated by introducing a negative additive constant in the potential. However, to make a reliable conclusions on the c-quark mass, we need improved measurements of $\Gamma(\psi \to \eta_c\gamma)$ with a better statistics. Such measurements are an important experimental goal, as the QCD sum rules predictions [16,17] (see table 2) also exceed the experimental data.

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Table 1. The ratios of the wave-function overlap integrals for the $\chi_0 \to \psi\gamma$, $\chi_2 \to \psi\gamma$ and $\psi' \to \chi_0\gamma, \psi' \to \chi_2\gamma$ transitions extracted from experiment[12] in comparison with the quark model predictions[4,6]: (a) the results extracted using nonrelativistic formulas, (b) the results extracted taking into account relativistic corrections by the formulas (32,33) and (35,36).

Overlap integral ratios	(a)	(b)	Quark model predictions	
			[4]	[6]
$\frac{r_{\psi\chi_0}}{r_{\psi\chi_2}}$	0.98 ± 0.23	1.06 ± 0.25	1.03	1.02
$\frac{r_{\chi_0\psi'}}{r_{\chi_2\psi'}}$	0.83 ± 0.05	0.77 ± 0.05	0.81	0.76

Table 2. Our results for $\Gamma(\psi \to \eta_c\gamma)$ in comparison with QCD sum rules predictions and experiment. In parentheses the nonrelativistic quark model predictions are given.

Our results		QCD sum rules	Experiment
$m = 1.6GeV$	$m = 1.8GeV$	$1.8 - 2.0KeV$ [16] $2.6 \pm 0.5KeV$ [17]	$1.14 \pm 0.3KeV$ [12]
$2.06KeV$ $(2.63KeV)$	$1.63KeV$ $(2.06KeV)$		